Refining volume estimates of down woody debris

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Abstract: Down woody debris (DWD) plays a vital role in forest ecosystem structure and function. Although volume is likely the most common metric used to characterize DWD, an evaluation of the formulae used for volume estimation on individual DWD pieces has received little attention. We determined actual volume of 155 diverse DWD pieces (types, species, lengths, and diameters) by detailed field measurements. By comparing the actual and calculated volumes from six commonly used formulae, we assessed their bias, precision, and accuracy. Based on observed DWD forms, we developed a new formula, namely the "conic–paraboloid", which was included in the assessment. Among the formulae that require length and two end diameter measurements, the conic–paraboloid had the lowest bias, highest precision, and hence greatest accuracy. Newton's and the centroid formulae had higher accuracy yet require more field measurements. Smalian's, conical frustum, and average-of-ends formulae had poor performance relative to the others. Accuracy of all formulae decreased with increasing piece length. Thus, partitioning pieces into two, three, and four sections for additional measurement improved accuracy. As decay advances, pieces become progressively more elliptical in cross section. Using the cross-sectional area derived from only the long axis of the ellipse leads to substantial volume overestimates for well-decayed DWD.

Résumé : Les débris ligneux jouent un rôle vital dans la structure et la fonction de l'écosystème forestier. Quoique le volume soit vraisemblablement la variable la plus généralement utilisée pour caractériser les débris ligneux, peu d'attention a été accordée à l'évaluation des formules utilisées pour estimer le volume de chaque pièce de débris ligneux. Nous avons déterminé le volume réel de 155 pièces de divers débris ligneux (types, essences, longueurs et diamètres) en prenant des mesures détaillées sur terrain. En comparant les volumes réels et estimés par six formules communément utilisées, nous avons calculé le biais, la précision et l'exactitude de ces formules. À partir d'observations sur la forme des débris ligneux, nous avons développé une nouvelle formule, la formule du paraboloïde conique, qui a également été incluse dans notre évaluation. Parmi les formules qui exigent la mesure de la longueur et du diamètre aux deux bouts, la formule du paraboloïde conique possède le biais le plus faible, est la plus précise et donc la plus exacte. Les formules de Newton et du centroïde sont plus exactes mais exigent plus de mesures sur le terrain. Les formules de Smalian, du tronc de cône et de la moyenne des diamètres aux extrémités performent moins bien que les autres. Quelle que soit la formule, l'exactitude décroît avec l'augmentation de la longueur de la pièce. Par conséquent, la subdivision des pièces en deux, trois ou quatre sections pour obtenir des mesures additionnelles améliore l'exactitude. À mesure que la décomposition des débris ligneux progresse, leur section transversale devient de plus en plus elliptique. L'utilisation de la surface radiale déterminée seulement à partir du grand axe de l'ellipse entraîne une surestimation substantielle du volume des débris ligneux dont la décomposition est avancée.

Introduction

Research over the past two decades has clearly demonstrated the importance of down woody debris (DWD) in forest ecosystem structure and function (see reviews in Harmon et al. 1986; McComb and Lindenmayer 1999; Siitonen 2001). Volume is perhaps the most common metric used to characterize DWD on a given site, providing a measure of substrate quantity for deadwood-dependent organisms, fuel loading for fire risk assessment, and structural diversity for geomorphological processes. Likewise, volume is the primary metric used when comparing DWD between sites, forest types, and forest disturbance histories. Field-measured volume is also essential for quantifying DWD biomass, a critical component of carbon budget modeling. Thus, an accurate method of determining DWD volume from field measurements is clearly needed within a broad range of ecological research.

Recent work has evaluated various field sampling protocols (e.g., plots, transects, line intercepts, and point relascopes) for estimating total DWD volume per unit area (e.g., Gove et al. 1999; Ringvall and Ståhl 1999; Ståhl et al. 2001; Woldendorp et al. 2004). These protocols rely on accurate volumes determined from individual DWD pieces within the area sampled. Determining the volume of individual pieces presents little challenge, provided that numerous measurements are obtained for each. However, given the large number of pieces typically surveyed in ecological studies, such detailed measurements become impractical. The challenge then is to estimate DWD volume using a small number of strategically placed measurements. Workers generally use one of the following six formulae to estimate volume: Newton's, Huber's, Smalian's, average-of-ends, centroid, and conical frustum (see Wiant et al. 1992; Fonweban 1997; Husch et al. 2003; Table 1).

We had assumed that the six formulae produced similar

Received 7 June 2006. Accepted 6 October 2006. Published on the NRC Research Press Web site at cjfr.nrc.ca on 29 May 2007.

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 doi:10.1139/X06-269 © 2007 NRC Canada

Table 1. Formulae for estimating down woody debris volume (V) including the conic–paraboloid introduced here.

Newton's	$V = \frac{L}{6}(A_b + 4A_m + A_u)$
Huber's	$V = LA_m$
Smalian's	$V = \frac{L}{2}(A_b + A_u)$
Conical frustum	$V = \frac{L}{3}(A_b + A_u + \sqrt{A_b A_u})$
Average-of-ends	$V = \frac{L\pi}{4}\left(\frac{D_b+D_u}{2}\right)^2$
Centroid method	Wiant et al. 1992; Patterson et al. 1993*a*
Conic–paraboloid (introduced here)	$V = \frac{L}{12}(5A_b + 5A_u + 2\sqrt{A_b A_u})$

Note: Given the complexity of the centroid method, we refer readers to outside sources. L represents piece length, A_b the cross-sectional area at the base, A_m the cross-sectional area at the longitudinal midpoint, A_u the cross-sectional area at the upper end, D_b the diameter at the base, and D_u the diameter at the upper end.

estimates; however, while merging and summarizing previously collected data from various laboratories, we found to our surprise that estimated volumes differed by as much as 47% for individual pieces and 38% for per-hectare estimates depending on which formulae were applied.

Given the obvious importance of tree volume in the commercial forestry sector, a large body of literature addresses volume and biomass estimation for individual tree species in particular regions (see reviews in Ter-Mikaelian and Korzukhin 1997; Zianis et al. 2005). These estimates form the basis of growth and yield modeling, forest productivity assessments, and economic forecasting. However, they have limited utility in typical field studies of DWD because (*i*) they are intended for intact trees, not the various bole segments, broken tops, and large branches typically encountered in field surveys, (*ii*) they generally estimate merchantable volume only, and (*iii*) they are based on diameter at breast height, which does not apply to broken stem sections, branches, or well-decayed pieces. Similarly, the numerous "scaling rules" or "log rules" used in commercial forestry are inappropriate because of their focus on merchantable volume.

An additional source of inaccuracy in DWD volume calculations results from the collapse of logs during the decay process. In fact, collapse is often used to assess decay in the myriad of existing decay stage classifications (e.g., McCullough 1948; Sollins 1982; Hofgaard 1993). In the field, however, workers typically measure the diameter of decayed logs, using calipers, along the long axis of the ellipse (i.e., parallel to the forest floor), given the difficulty of measuring the short-axis diameter (i.e., perpendicular to the forest floor), especially when decayed logs are imbedded in the substrate. Using the cross-sectional area derived from only the long-axis diameter clearly overestimates the current volume of well-decayed DWD.

The purpose of this study was thus twofold. First, we aimed to evaluate the bias, precision, and accuracy of the six commonly used formulae for DWD volume estimation, with the intent of determining which is most appropriate for use in field protocols. Although several of these formulae have been similarly assessed for intact tree boles after harvesting, to the best of our knowledge, this is the first attempt to do so for the diverse DWD forms encountered during ecological field surveys. In doing so, we developed a new formula that has relatively high accuracy and ease of use in the field. Second, we aimed to demonstrate the degree to which collapse, if not taken into account, could influence volume estimates of well-decayed DWD.

The formulae

We present here an overview of the six common formulae (see Table 1); additional detail can be found in Wiant et al. (1992), Patterson et al. (1993*a*), Fonweban (1997), and Husch et al. (2003). All formulae can be applied to frusta (truncated forms of rotated solids). All require length measures but differ according to the number and placement of diameter measures. Huber's formula requires only one diameter taken at the longitudinal midpoint; the remaining formulae require diameter measures at the large and small ends. Newton's formula requires an additional diameter at the longitudinal midpoint, and the centroid method requires a diameter taken at the center of volume, once calculated. The centroid can more properly be considered a method, not merely a formula, and is rather complex relative to the others (Wiant et al. 1992; Patterson et al. 1993*a*). Smalian's and Huber's formulae assume that the piece has the form of a second-order paraboloid, Newton's formula applies to paraboloid, neiloid, or conical forms, the average-of-ends formula assumes the form of a cylinder, and the conical frustum of course assumes the form of a cone. The three "surname" formulae have been in use a surprisingly long time: Newton's is likely attributable to Sir Issac Newton, Huber's first appeared in 1785, and Smalian's simplified formula came into use in 1894 (Prodan 1965, cited in Husch et al. 2003). Fonweban (1997) pointed out that the average-of-ends formula may be more commonly used than is thought, as it may at times be mistakenly referred to as Huber's formula.

While viewing stem-form graphs of all pieces, we recognized that the typical form was neither conic nor second-order parabolic but rather somewhere in between the two. Graphs suggested that the conic form would consistently underestimate and a second-order paraboloid consistently overestimate volume (see Fig. 1), which was ultimately borne out by the analyses. We thus simply combined the conical frustum and Smalian's formulae, giving each equal weight, to produce what we refer to as the "conic–paraboloid" formula, which is presented in Table 1. We evaluated its performance along with the six common existing formulae.

Methods

We made detailed measurements on relatively nonde-

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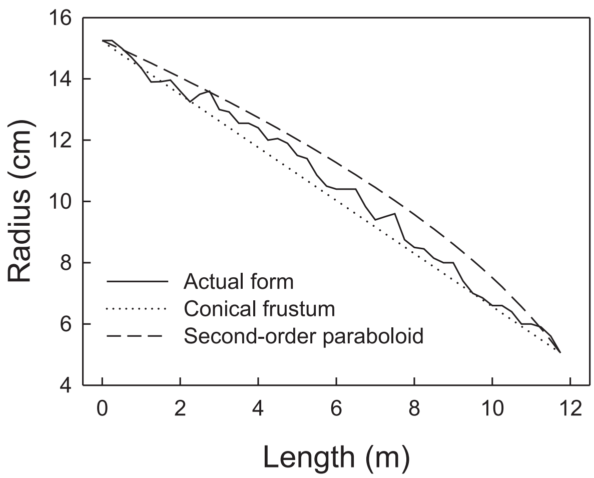
Fig. 1. Stem form for one selected woody debris piece (with radii at every 25 cm increment) showing that the conical frustum underestimates and the second-order paraboloid (assumed by Smalian's formula) overestimates the actual volume. These graph forms motivated the conic–paraboloid formula introduced here, which combines the conical frustum and second-order paraboloid formulae. Vertical axis exaggerated for presentation.



cayed Norway spruce (*Picea abies* (L.) Karst.) (n = 65), Scots pine (*Pinus sylvestris* L.) (n = 45), and downy birch (*Betula pubescens* Ehrh.) (n = 45) DWD pieces located at numerous sites in north-central Sweden (centered at 62°30′N, 16°30′E). Mean annual temperature is approximately 3 °C, annual precipitation is approximately 700 mm, and the mean site elevation is approximately 200 m above sea level. All sites supported mature stands dominated by Norway spruce or Norway spruce – Scots pine (various proportions), with downy birch forming a lesser component. These stands form closed canopies at maturity, meaning that stem growth form is largely monopodial. DWD pieces were selected to span a range of large-end diameters, with pieces fairly evenly spread across the following diameter classes (as midpoints): 12.5, 17.5, 22.5, 27.5, 32.5, 37.5, and 42.5+ cm (but 37.5+ cm for downy birch). Pieces were also selected to cover the range of deadwood types: uprootings, wind snaps, bole segments (either broken naturally or left after harvesting), broken tops, and logging cull. The mean and range of large-end diameters, lengths, and volumes are presented in Table 2.

An evaluation of the formulae required that we determine the actual volume (outside bark by convention) of each DWD piece. Beginning at the large end, we measured diameters (with calipers to the nearest millimetre) at 25 cm intervals along the entire length. We then calculated actual volume using two methods for comparison. First, for each piece, we determined the cross-sectional area at each measured point, fit a flexible cubic spline to the areas, and integrated the spline to arrive at total volume. Second, given these short intervals, we assumed that each 25 cm segment had the shape of a conical frustum and summed segment volumes accordingly. In no case did the two methods produce total volumes that differed by more than 0.00155 m^3 (mean percentage differences 0.031%, maximum 0.75%). Results from the latter method were taken as actual volumes throughout the study.

On pieces with pronounced basal flare (i.e., "butt swell", common on uprootings), we used the diameter just above the flare as the large-end diameter in formulae evaluation, as using the extreme flared diameter would result in substantial overestimates. From our combined field experience, this practice seems entirely reasonable, and we assume that other workers do the same. We return to this topic in the Results and discussion section.

We assessed the bias, precision, and accuracy of each formula by comparing calculated and actual volumes. Bias was estimated as the mean difference between the calculated and actual volumes, precision was estimated as the standard deviation (SD) of the differences, and accuracy was evaluated by the mean square error, i.e., variance + bias^2 (Cochran 1977; Fonweban 1997). To express accuracy in the same units as those measured, we converted to the root mean square error. The ideal formula would have bias near zero, high precision (i.e., low SD), and high accuracy (i.e., low root mean square error).

Our data also allowed us to evaluate the benefit of "sectioning" (i.e., partitioning the piece into two or more components, with measures taken on each; see Ståhl et al. 2001) as a means of increasing accuracy of DWD volume estimates. We thus partitioned each DWD piece into two, three, and four sections, estimating the volume of each by the average-of-ends, conical frustum, Smalian's, and conic–paraboloid formulae. Estimated section volumes were then summed to arrive at a presumably more accurate estimated total volume. We limited this analysis to these two-diameter formulae because of their flexibility in field applications: diameters could be taken at haphazard, convenient, or randomly chosen locations along the piece length, thereby defining the section divisions, without the need to calculate section midpoints. However, for convenience in programming and summarizing our own data, we divided pieces into sections of nearly equal length. Recognizing that sectioning would only be applied to long pieces, where its benefit would be most pronounced, we also limited this analysis to pieces over 10 m in length. We note that our method of determining actual volume is an extreme example of sectioning.

To assess DWD collapse in advanced stages of decay, we measured width (parallel to forest floor) and height (perpendicular to forest floor) on log cross sections carefully exposed by sawing and excavation (when necessary). These measurements were made on ≥25 Norway spruce logs in each of the four most advanced stages (5–8) in our eight-stage system of decay described as follows: (5) wood soft, with crevices and pieces lost, large branches remain, form may be elliptical in cross-section, (6) wood soft, larger pieces lost, branch stubs pull out easily, form elliptical in cross section, (7) wood very soft and easily crumbled but possibly with a core of harder wood, form clearly elliptical in cross section, and (8) wood well decayed, form elliptical, flattened, or sunken into the substrate.

Results and discussion

The formulae can be grouped conveniently according to the number of field measurements needed. Newton's and the centroid formulae require five measures (three diame-

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Table 2. Means, minima, and maxima for various measures of down woody debris sampled.

		Large-end diameter (cm)			Length (m)			Volume (m^3)		
Species	*n*	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
Norway spruce (*Picea abies*)	65	26.5	10.4	57.9	11.85	3.00	29.25	0.455	0.019	2.862
Scots pine (*Pinus sylvestris*)	45	25.7	10.7	51.0	10.56	2.75	20.75	0.318	0.018	1.268
Downy birch (*Betula pubescens*)	45	22.2	10.0	42.7	8.31	2.25	22.75	0.258	0.015	1.592

ters, length, and an intermediate longitudinal measure), while the remaining formulae require three measures (Table 3). When considered across all pieces, Newton's and the centroid formulae outperformed the others with regard to bias and precision and hence accuracy (Table 3). These two formulae had nearly identical precision; however, Newton's had slightly lower bias, making it the most accurate formula assessed. Among the three-measurement formulae, Huber's and the conic–paraboloid outperformed the remainders. These two differed with respect to bias (conic–paraboloid preferred) and precision (Huber's preferred), with Huber's having higher overall accuracy. Our results reveal relatively low accuracy for the average-of-ends, conical frustum, and Smalian's formulae (Table 3). Perhaps paradoxically, the latter two are among the more widely used, including our own previous studies (e.g., Jonsson 2000; Fraver and White 2005). We note that precision is rather low (i.e., high SD, Table 3) even for the better formula, indicating that the difference between actual and calculated volume can still be relatively high on a given DWD piece. However, for estimates of the aggregated volume of many pieces in an area, the effect of the SD on the error of the final estimate will decrease by the factor $n^{-1/2}$, where n is the number of sampled pieces. In contrast, bias does not decrease with increasing sample size.

Although formulae have not been previously evaluated for diverse deadwood types, several studies of recently harvested, lower bole segments are available for comparison. Young et al. (1967) reported results similar to ours, with Newton's formula having bias near 0%, Huber's –3.5% bias on average, and Smalian's 9% bias on average based on conifer boles up to 16 ft (4.9 m) in length. Fonweban (1997) found Newton's formula to be the most accurate, Huber's and average-of-ends intermediate in accuracy, and Smalian's the least accurate based on entire boles from three tropical hardwood species. In contrast, Figueiredo Filho et al. (2000) found Huber's formula to be the most accurate of six formulae evaluated, including Newton's, Smalian's, and the centroid. Their work was conducted on short basal segments (to 6 m) of plantation-grown slash pine (*Pinus elliottii* Engelm.). Patterson et al. (1993*a*), Patterson et al. (1993*b*), and Wiant et al. (1996) found that the centroid method outperformed Newton's, Huber's, and Smalian's methods based on lower bole segments of various species in the southeastern United States. The overall bias of Newton's, Huber's, and Smalian's formulae from the present study is surprisingly similar to that proposed by Husch et al. (2003), with the bias from Newton's being near zero and that of Huber's being opposite in sign and roughly one half that of Smalian's (Table 3).

These published results, coupled with a careful reading of the literature, reveal that previous workers have recognized, either explicitly or implicitly, that the typical stem form lies between a cone and a second-order paraboloid (Fig. 1). Smalian (1837, cited in Bruce 1987) derived a general volume formula for a frustum of a rotated solid, allowing for varying shape parameters that determine the degree of stem convexity, which could range from conic to higher order paraboloid. The commonly used Smalian's formula is a simplified version of the original, assuming the form of a second-order paraboloid. Forslund (1982) described a geometrical model for entire trembling aspen (*Populus tremuloides* Michx.) trees with the stem form between a cone and a paraboloid, referring to the form as a "paracone". Husch et al. (2003) pointed out that general stem forms can be described by rotating the curve $y^2 = kx^r$ around the x-axis. Here, y is the radius a distance x from the top of the (entire) tree, r is a shape parameter that determines stem form ($r = 2$ produces a cone and $r = 1$ produces a paraboloid), and k is the taper rate. Expressed in this form, Forslund's (1982) paracone would have a shape parameter of 4/3 (Lynch et al. 1994), i.e., intermediate between a cone and a paraboloid. Substituting this shape parameter into Smalian's (1837) equation and applying it to our data produced volume estimates with equal precision but slightly higher bias when compared with our conic–paraboloid formula. Smalian's (1837) original equation, however, is likely too complex to receive widespread use in studies of DWD, and Forslund's model applies only to entire trees, making it inappropriate for such studies. Taken together, this previous work lends support for the conic–paraboloid formula introduced here. In summary, although it has long been recognized that the typical stem form often lies between a cone and a second-order paraboloid, ours is the first attempt to provide a simple and accurate formula based on this recognition.

In designing this study, we had not intended to compare formula performance between length classes, species, or DWD types. Instead, our objective was to determine if one formula could be applied, with acceptable accuracy, across the diverse pieces encountered in field inventories. Nevertheless, some of our results regarding such data subsets are perhaps worth noting here. For all formulae, accuracy decreased substantially with increasing piece length (Table 3), a finding that is well reported in the literature (e.g., Patterson et al. 1993*b*; Fonweban 1997; Figueiredo Filho et al. 2000). For the two longest classes, the ranking of formula remained unchanged from that of the pooled data. In contrast, for the shortest class, the ranking was disrupted. However, when applied to short pieces, all formula had relatively high accuracy, suggesting that formula selection has less bearing on shorter than on longer pieces. Similarly, when separated by species, the ranking of formula remained relatively constant (Table 3). The only notable exception is the performance of the centroid method, which for downy birch represented an improvement in ranking (making the centroid

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Table 3. Evaluation of formulae for estimating down woody debris volume ranked by decreasing accuracy (i.e., increasing root mean square error (RMSE)).

	Field measures		All pieces				RMSE by length class (m)			RMSE by species		
Formula	Diameter	Intermediate	RMSE	Bias	%bias	SD	<10	10–20	>20	Norway spruce (*Picea abies*)	Scots pine (*Pinus sylvestris*)	Downy birch (*Betula pubescens*)
Newton	3	1	0.026	0.002	0.6	0.026	0.014	0.031	0.048	0.030	0.022	0.026
Centroid	3	1	0.028	–0.011	–2.9	0.026	0.012	0.031	0.059	0.033	0.030	0.020
Huber	1	1	0.048	–0.019	–5.2	0.045	0.022	0.050	0.104	0.065	0.024	0.038
Conic–paraboloid	2	0	0.056	0.008	2.2	0.055	0.025	0.060	0.117	0.068	0.027	0.058
Conical frustum	2	0	0.071	–0.027	–7.6	0.065	0.022	0.066	0.166	0.078	0.066	0.064
Smalian	2	0	0.104	0.043	12.0	0.094	0.030	0.107	0.235	0.130	0.065	0.094
Average-of-ends	2	0	0.128	–0.062	–17.4	0.112	0.019	0.117	0.313	0.146	0.123	0.105

Note: Field measures refer to the number diameter and intermediate longitudinal measurements needed for each formula. Percent bias refers to the average residual volume (calculated minus actual) expressed as a percentage of the average actual volume. Lower standard deviations (SD) indicate higher precision.

the most accurate) but for Scot's pine representing a reduction (making it the fourth most accurate). Any further comparisons between species or types are confounded by the strong effect of piece length because lengths differed somewhat between species and types (e.g., Table 2).

Given that accuracy increases with decreasing length (Table 3), the benefits of sectioning of pieces for additional measurements can clearly be seen (Fig. 2). Division of pieces into two sections provides the most dramatic improvement, reducing bias and precision to ranges likely acceptable in most studies. For example, double sectioning using the conic–paraboloid produces bias and precision nearly identical to those of Newton's formula applied to nonsectioned pieces (Table 3; Fig. 2). While sectioning into three pieces may be desirable for certain studies, sectioning beyond three pieces seems to provide little additional benefit (Fig. 2). We point out that sectioning into two pieces using formulae shown in Fig. 2 requires the same number of field measurements as applying Newton's or the centroid method to nonsectioned pieces, but without the need for intermediate length calculations.

Basal flare on lower bole segments (i.e., butt swell) has long complicated volume estimation in commercial forestry. Although the volume of the basal flare can be estimated by the formula for a neiloid (Husch et al. 2003), obtaining an estimate of the total log volume would require partitioning the piece into two sections for measurement, with separate formulae applied to each. Alternately, Bruce (1982) and Patterson and Doruska (2004) have proposed modifications to Smalian's formula for use on such butt logs in an attempt to reduce bias and increase accuracy. To the best of our knowledge, this issue has not been addressed in studies of DWD, perhaps because flared basal pieces represent only one of many DWD forms typically encountered and because the original stem form cannot be easily ascertained for well-decayed pieces. Instead, workers apply one volume formula to all DWD pieces (but see Busing 2005). Consistent with this practice, we have not attempted to treat pieces with basal flare differently from others. For such pieces, we used in our calculations the large-end diameter immediately above the flare (see Methods section), producing a slight underestimate of actual volume on flared pieces (mean = 0.0117, median = 0.0075, maximum = 0.0568 m^3).

Selecting a formula for a particular study represents a balance between accuracy and field efficiency. The centroid method, although quite accurate, has the disadvantage of requiring a programmable calculator or handheld personal computer to determine the position at which to measure center-of-volume diameter once length and two end diameters have been measured. Both Huber's and Newton's formulae require field calculation of the longitudinal midpoint, which adds time and introduces a potential source of error. Perhaps the most efficient methods are those that require one length and two end diameters. Among the four such formulae, the conic–paraboloid had the lowest bias, highest precision, and hence greatest accuracy (Table 3).

Figure 3 presents the mean collapse ratios (height to width) for the four most advanced decay stages, clearly showing greater collapse and higher variability in the ratio as decay advances. We present these ratios primarily to illustrate a point that may be overlooked in studies of DWD.

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Fig. 2. The benefit of sectioning (partitioning the piece into two or more components for measurement) with regard to standard deviation (an inverse measure of precision) and bias.

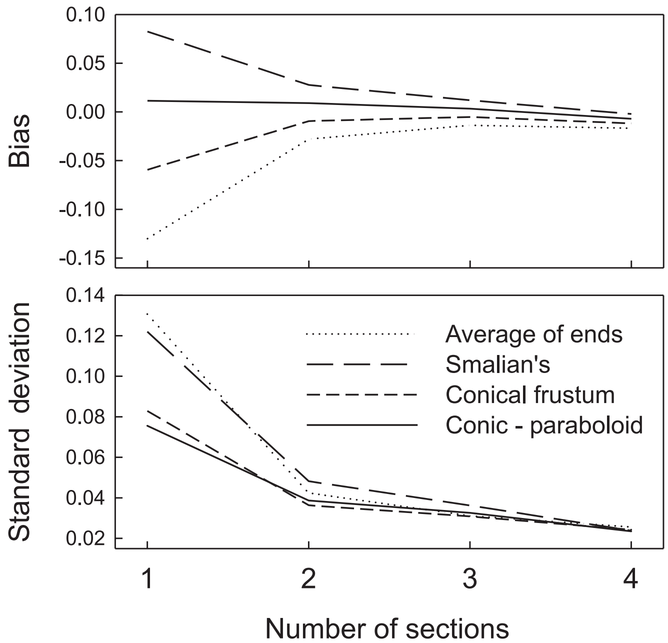
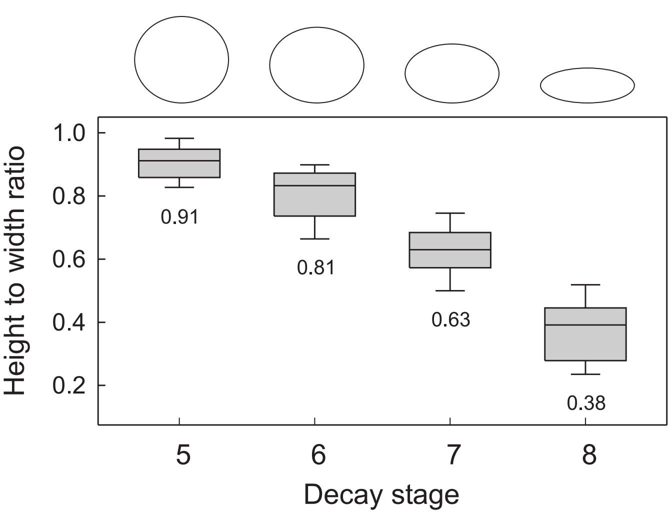


Fig. 3. Collapse ratios (height to width) for the four most advanced stages of wood decay (our eight-stage system) for Norway spruce (*Picea abies*). Assuming circular cross sections when applying the volume formulae can lead to dramatic overestimation for advanced decay stages. The midline of each box represents the median ratio, box boundaries the 25th and 75th percentiles, and the whiskers the 10th and 90th percentiles. Mean ratios are provided below each box. Ellipses above the graph illustrate cross sections adhering to each ratio.



If the intent is to estimate biomass or carbon stocks, to estimate fuel loads, or to quantify substrate available for deadwood-dependent organisms, then the elliptical shape of well-decayed logs should be taken into account to avoid substantial overestimation (e.g., by approximately 9%, 19%, 37%, and 62% in decay stages 5–8, respectively) (Fig. 3). Such adjustments appear to be rarely applied (but see Spies et al. 1988; Fraver et al. 2002). Of course, because wood density decreases with increasing decay, it too must be taken into account for biomass or carbon stock estimation. If the intent, however, is to estimate the volume of fresh wood added in the past (useful in studies of forest disturbance), then a circular shape can be assumed as follows. Given that volume loss results primarily from collapse, log width does not change substantially through decay. The roughly triangular or trapezoidal outline of a log, when viewed from above, remains relatively constant through all but the most advanced stages of decay. Thus, the current width, measured parallel to the forest floor, provides a reasonable estimate of the fresh diameter and hence volume. Obvious exceptions are pieces fragmented by mechanical force.

We note also that our collapse ratio determined for the most advanced decay stage includes considerable uncertainty given that only decayed logs visible as mounds (covered with ground-layer vegetation) were sampled. The most advanced (i.e., flattened) pieces were hidden under the ground-layer vegetation and remained unsampled. Even when encountered, these pieces present a sampling dilemma because the most advanced decay stage has no clearly defined endpoint separating decayed wood from organic soil.

Recommendations

Given the range of estimated volumes produced from the formulae evaluated here, we recommend at a minimum that workers present the formula used, a detail often overlooked in published work. Where accuracy is critical, Newton's or the centroid formula should be used, as both provide high precision and minimal bias. Similar accuracy can be achieved by double or triple sectioning of long pieces, preferably using the conic–paraboloid formula introduced here. Sectioning into more than three pieces appears to provide little added benefit. For ease of use in the field, as well as reasonable performance on nonsectioned pieces, we recommend the conic–paraboloid formula, assuming that a bias of approximately 2% is acceptable. The conical frustum provides a reasonable alternative (approximately –4% bias, but with much lower precision). We recommend against the use of Smalian's (approximately 12% bias) or the average-of-ends (approximately –17% bias) formula unless accompanied by sectioning. If the current volume of well-decayed pieces is needed, we recommend that the decay-related collapse be taken into account to avoid overestimates arising from field measurements of only log widths.

Acknowledgements

We thank Brittany Armstrong-Gordon and Mari Jönsson for assistance in the field and Per Edström for mathematical advice. Thoughtful comments from two anonymous reviewers substantially improved the manuscript. Financial support was provided by the Municipality of Sundsvall, Sweden. A.R. was partly funded by the Heureka Research Program at the Swedish University of Agricultural Sciences and by the Swedish Research Council for Environment, Agricultural Sciences, and Spatial Planning (FORMAS).

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